

## DIE AUFGABEN

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$$1 \quad \left( \frac{1}{r-s} - \frac{1}{r+s} \right)^2 =$$

$$2 \quad \frac{x^4}{46y^3} : \left( \frac{x^2}{23y} : \frac{5y}{x^2} \right) =$$

$$3 \quad \frac{a^2 - 13a - 230}{10} : \left( \frac{a}{100} + \frac{1}{10} \right) =$$

$$4 \quad \left( \frac{ab}{a-b} + a \right) \left( \frac{ab}{a+b} - b \right) \cdot \frac{b-a}{ab^2} =$$

$$5 \quad \frac{3a}{3a-2b} \cdot \frac{3a}{2b} - \left( \frac{3a}{3a-2b} + \frac{3a}{2b} \right) =$$

$$6 \quad \left( \frac{8x^2 + 4x + 1}{4x^2 - 2x} - \frac{2x}{2x-1} \right) \frac{6x-3}{4x^2 + 2x} =$$

$$7 \quad \left( \frac{n^3 - 2n - 1}{n^2 - 1} - n \right) \left( n - \frac{2n^2}{n+1} \right) =$$

$$8 \quad \left( x - \frac{1}{x} \right) : \left( x + \frac{1}{x} \right) =$$

$$9 \quad \frac{72(a-b)^2}{25a^3} \left( \frac{54c^3}{(a-b)^3} : \frac{81c^2}{5a^2} \right) =$$

$$10 \quad \left( \frac{2a+1}{2a-1} - \frac{2a-1}{2a+1} \right) \cdot \left( \frac{a}{2} - \frac{1}{2} + \frac{1}{8a} \right) =$$

$$11 \quad \left( \frac{a^3 - 1}{a^3} - \frac{a^2 - a - 1}{a^2} - \frac{1}{a} \right) : \frac{1}{a^3} =$$

$$12 \quad \frac{a^2b}{8} \left\{ \left( \frac{1}{a} - \frac{1}{b} \right) \left( \frac{1}{a} - \frac{1}{b} + \frac{1}{c} \right) + \left( \frac{1}{b} - \frac{1}{a} + \frac{1}{c} \right) \left( \frac{1}{b} - \frac{1}{a} \right) \right\} =$$

## DIE LÖSUNGEN

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$$1 \quad \left( \frac{1}{r-s} - \frac{1}{r+s} \right)^2 = \left( \frac{r+s - (r-s)}{(r-s)(r+s)} \right)^2 = \left( \frac{2s}{r^2 - s^2} \right)^2 = \frac{4s^2}{(r^2 - s^2)^2}$$

Ich beginne hier mit der Subtraktion, weil das Resultat relativ einfach ist.

$$2 \quad \frac{x^4}{46y^3} : \left( \frac{x^2}{23y} : \frac{5y}{x^2} \right) = \frac{x^4}{46y^3} : \left( \frac{x^2}{23y} \cdot \frac{x^2}{5y} \right) = \frac{x^4}{46y^3} : \frac{x^4}{5 \cdot 23y^2} = \frac{x^4}{46y^3} \cdot \frac{5 \cdot 23y^2}{x^4} = \frac{5}{2y}$$

Klammerrechnung zuerst!

$$3 \quad \frac{a^2 - 13a - 230}{10} : \left( \frac{a}{100} + \frac{1}{10} \right) = \frac{(a-23)(a+10)}{10} : \frac{a+10}{100} = \frac{(a-23)(a+10)}{10} \cdot \frac{100}{a+10} = 10(a-23)$$

$$4 \quad \left( \frac{ab}{a-b} + a \right) \left( \frac{ab}{a+b} - b \right) \cdot \frac{b-a}{ab^2} = \left( \frac{ab + a(a-b)}{a-b} \right) \left( \frac{ab - b(a+b)}{a+b} \right) \cdot \frac{b-a}{ab^2} \\ = \frac{a^2}{a-b} \cdot \frac{-b^2}{a+b} \cdot \frac{b-a}{ab^2} = \frac{-a(b-a)}{(a-b)(a+b)} = \frac{a(-b+a)}{(a-b)(a+b)} = \frac{a}{a+b}$$

$$5 \quad \frac{3a}{3a-2b} \cdot \frac{3a}{2b} - \left( \frac{3a}{3a-2b} + \frac{3a}{2b} \right) = \frac{9a^2}{2b(3a-2b)} - \frac{6ab + 3a(3a-2b)}{2b(3a-2b)} \\ = \frac{9a^2 - (6ab + 9a^2 - 6ab)}{2b(3a-2b)} = \frac{0}{2b(3a-2b)} = 0$$

$$6 \quad \left( \frac{8x^2 + 4x + 1}{4x^2 - 2x} - \frac{2x}{2x-1} \right) \cdot \frac{6x-3}{4x^2 + 2x} = \left( \frac{8x^2 + 4x + 1}{2x(2x-1)} - \frac{2x}{2x-1} \right) \cdot \frac{6x-3}{4x^2 + 2x} \\ = \frac{8x^2 + 4x + 1 - 4x^2}{2x(2x-1)} \cdot \frac{3(2x-1)}{2x(2x+1)} = \frac{4x^2 + 4x + 1}{2x} \cdot \frac{3}{2x(2x+1)} \\ = \frac{(2x+1)^2 \cdot 3(x-2)}{4x^2(2x+1)} = \frac{3(2x+1)}{4x^2}$$

$$\begin{aligned}
7 \quad \left( \frac{n^3 - 2n - 1}{n^2 - 1} - n \right) \left( n - \frac{2n^2}{n+1} \right) &= \frac{n^3 - 2n - 1 - n(n^2 - 1)}{n^2 - 1} \cdot \frac{n(n+1) - 2n^2}{n+1} \\
&= \frac{n^3 - 2n - 1 - n^3 + n}{n^2 - 1} \cdot \frac{n^2 + n - 2n^2}{n+1} = \frac{-n-1}{n^2 - 1} \cdot \frac{n-n^2}{n+1} \\
&= \frac{-1 \cdot (n+1) \cdot (-n) \cdot (-1+n)}{\cancel{(n-1)}(n+1) \cdot \cancel{(n+1)}} = \frac{n}{n+1}
\end{aligned}$$

$$8 \quad \left( x - \frac{1}{x} \right) : \left( x + \frac{1}{x} \right) = \frac{x^2 - 1}{x} : \frac{x^2 + 1}{x} = \frac{x^2 - 1}{x} \cdot \frac{x}{x^2 + 1} = \frac{x^2 - 1}{x^2 + 1}$$

Divisionen durch eine Summe müssen meist so gelöst werden!

$$\begin{aligned}
9 \quad \frac{72(a-b)^2}{25a^3} \left( \frac{54c^3}{(a-b)^3} : \frac{81c^2}{5a^2} \right) &= \frac{72(a-b)^2}{25a^3} \left( \frac{\cancel{54}c^3}{(a-b)^3} \cdot \frac{5a^2}{\cancel{81}c^2} \right) \\
&= \frac{72(a-b)^2}{25a^3} \cdot \frac{2c \cdot 5a^2}{(a-b)^3 \cdot 3} = \frac{48c}{5a(a-b)}
\end{aligned}$$

$$\begin{aligned}
10 \quad \left( \frac{2a+1}{2a-1} - \frac{2a-1}{2a+1} \right) \cdot \left( \frac{a}{2} - \frac{1}{2} + \frac{1}{8a} \right) &= \frac{(2a+1)^2 - (2a-1)^2}{(2a+1)(2a-1)} \cdot \frac{4a^2 - 4a + 1}{8a} \\
&= \frac{4a^2 + 4a + 1 - (4a^2 - 4a + 1)}{(2a+1)(2a-1)} \cdot \frac{(2a-1)^2}{8a} \\
&= \frac{8a}{(2a+1)(2a-1)} \cdot \frac{(2a-1)^2}{8a} = \frac{2a-1}{2a+1}
\end{aligned}$$

$$\begin{aligned}
11 \quad \left( \frac{a^3 - 1}{a^3} - \frac{a^2 - a - 1}{a^2} - \frac{1}{a} \right) : \frac{1}{a^3} &= a^3 \cdot \left( \frac{a^3 - 1}{a^3} - \frac{a^2 - a - 1}{a^2} - \frac{1}{a} \right) \\
&= (a^3 - 1) - a(a^2 - a - 1) - a^2 = a^3 - 1 - a^3 + a^2 + a - a^2 = a - 1
\end{aligned}$$

$$\begin{aligned}
12 \quad \frac{a^2b}{8} \left\{ \left( \frac{1}{a} - \frac{1}{b} \right) \left( \frac{1}{a} - \frac{1}{b} + \frac{1}{c} \right) + \left( \frac{1}{b} - \frac{1}{a} + \frac{1}{c} \right) \left( \frac{1}{b} - \frac{1}{a} \right) \right\} \\
&= \frac{a^2b}{8} \left\{ \frac{1}{a^2} - \frac{1}{ab} + \frac{1}{ac} - \frac{1}{ab} + \frac{1}{b^2} - \frac{1}{bc} + \frac{1}{b^2} - \frac{1}{ab} - \frac{1}{ab} + \frac{1}{a^2} + \frac{1}{bc} - \frac{1}{ac} \right\} \\
&= \frac{a^2b}{8} \left\{ \frac{2}{a^2} - \frac{4}{ab} + \frac{2}{b^2} \right\} = \frac{a^2b}{8} \cdot \frac{2b^2 - 4ab + 2a^2}{a^2b^2} = \frac{2(b^2 - 2ab + a^2)}{8b} = \frac{(a-b)^2}{4b}
\end{aligned}$$