

a) $x^2 - 2ax + 6ab = 9b^2$

$$x^2 - 2ax + 6ab - 9b^2 = 0$$

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$$\begin{aligned}x_{1,2} &= \frac{2a \pm \sqrt{4a^2 - 4(6ab - 9b^2)}}{2} \\ &= \frac{2a \pm \sqrt{4a^2 - 24ab + 36b^2}}{2} = \frac{2a \pm \sqrt{(2a - 6b)^2}}{2} = \frac{2a \pm (2a - 6b)}{2}\end{aligned}$$

$$x_1 = \frac{2a + (2a - 6b)}{2} = \frac{4a - 6b}{2} = 2a - 3b$$

$$x_2 = \frac{2a - (2a - 6b)}{2} = \frac{6b}{2} = 3b$$

b) $x^2 - x + a = a^2$

$$x^2 - x + a - a^2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 - 4(a - a^2)}}{2} = \frac{1 \pm \sqrt{1 - 4a + 4a^2}}{2} = \frac{1 \pm \sqrt{(1 - 2a)^2}}{2} = \frac{1 \pm (1 - 2a)}{2}$$

$$x_1 = \frac{1 + 1 - 2a}{2} = \frac{2 - 2a}{2} = 1 - a$$

$$x_2 = \frac{1 - 1 + 2a}{2} = \frac{2a}{2} = a$$

c) $x^2 - b^2 = a(2x - a)$

$$\begin{aligned}x^2 - b^2 &= a(2x - a) \\x^2 - b^2 &= 2ax - a^2 \\x^2 - 2ax + a^2 - b^2 &= 0\end{aligned}$$

$$x_{1,2} = \frac{2a \pm \sqrt{4a^2 - 4(a^2 - b^2)}}{2} = \frac{2a \pm \sqrt{4b^2}}{2} = \frac{2a \pm 2b}{2} = a \pm b$$

$$\mathbf{x_1 = a + b}$$

$$\mathbf{x_2 = a - b}$$

d) $(a^2 - b^2)x^2 - 2ax + 1 = 0$

$$(a^2 - b^2)x^2 - 2ax + 1 = 0$$

$$x_{1,2} = \frac{2a \pm \sqrt{4a^2 - 4 \cdot (a^2 - b^2) \cdot 1}}{2(a^2 - b^2)} = \frac{2a \pm \sqrt{4b^2}}{2(a^2 - b^2)} = \frac{2a \pm 2b}{2(a^2 - b^2)} = \frac{2(a \pm b)}{2(a+b)(a-b)}$$

$$\mathbf{x_1 = \frac{1}{a-b}}$$

$$\mathbf{x_2 = \frac{1}{a+b}}$$