$$\int \frac{\sqrt{\ln x}}{x}\,dx$$

Substitution:

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x} \qquad \Big| \cdot dx$$

$$du = \frac{dx}{x}$$

$$\int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{\ln x} \cdot \frac{dx}{x} = \int \sqrt{u}\,du = \int u^{0.5}\,du = \frac{u^{1.5}}{1.5} = \frac{2\sqrt{u^3}}{3} = \frac{2\sqrt{(\ln x)^3}}{3}$$

$$\int \frac{x^2 - 1}{x^2 + 1}\,dx$$

Kunstgriff:

$$\int \frac{x^2 - 1}{x^2 + 1}\,dx = \int \frac{x^2 + 1 - 2}{x^2 + 1}\,dx = \int \left(\frac{x^2 + 1}{x^2 + 1} - \frac{2}{x^2 + 1} \right) dx = \int \left(1 - \frac{2}{x^2 + 1} \right) dx$$

$$= \int 1\,dx - \int \frac{2}{x^2 + 1}\,dx$$

Das 2. Integral haben wir schon in Nummer 5a gelöst.

$$\int \frac{x^2 - 1}{x^2 + 1}\,dx = x - 2\arctan x$$