

$$(\sqrt{a+b} + \sqrt{a-b})^2 - (\sqrt{a+b} - \sqrt{a-b})^2$$

Der obige Term hat vereinfacht die Form:

$$x^2 - y^2 \quad \text{wobei} \quad x = \sqrt{a+b} + \sqrt{a-b} \quad \text{und} \quad y = \sqrt{a+b} - \sqrt{a-b}$$

$$\text{Nun gilt aber: } x^2 - y^2 = (x+y)(x-y)$$

$$x+y = (\sqrt{a+b} + \sqrt{a-b}) + (\sqrt{a+b} - \sqrt{a-b}) = 2\sqrt{a+b}$$

$$x-y = (\sqrt{a+b} + \sqrt{a-b}) - (\sqrt{a+b} - \sqrt{a-b}) = \sqrt{a+b} + \sqrt{a-b} - \sqrt{a+b} + \sqrt{a-b} = 2\sqrt{a-b}$$

und

$$(x+y)(x-y) = 2\sqrt{a+b} \cdot 2\sqrt{a-b} = 4\sqrt{(a+b)(a-b)} = 4\sqrt{a^2 - b^2}$$