Sechs einfache Exponentialgleichungen, bei denen beide Seiten der Gleichung logarithmiert werden müssen:

a) $2^{3x} = 5$ b) $5^{3x-2} = 7$ c) $2^{x-2} = 5^{x-1}$

d) $\frac{1}{3^x} = 4$ e) $\sqrt[x]{20} = 5$ f) $3^{x-1} \cdot 2^{2x} = 5^{3x+1}$

Kontrollieren Sie die Lösungen, indem Sie sie im Taschenrechner berechnen, unter x speichern und dieses x in der Ausgangsgleichung einsetzen!

a)
$$\log(2^{3x}) = \log 5$$
$$3x \cdot \log 2 = \log 5$$
$$x = \frac{\log 5}{3\log 2} = \frac{\log 5}{\log 2^3}$$
$$\mathbf{x = \frac{log5}{log8} \approx 0.774}$$

b)
$$5^{3x-2} = 7$$
$$\log(5^{3x-2}) = \log 7$$
$$(3x-2)\log 5 = \log 7$$
$$3x\log 5 - 2\log 5 = \log 7$$
$$3x\log 5 = \log 7 + 2\log 5$$
$$x\log 5^3 = \log(7 \cdot 5^2)$$
$$x\log 125 = \log 175$$
$$\mathbf{x = \frac{log175}{log125} \approx 1.070}$$

c)
$$2^{x-2} = 5^{x-1}$$
$$\log(2^{x-2}) = \log(5^{x-1})$$
$$(x-2)\log 2 = (x-1)\log 5$$
$$x\log 2 - 2\log 2 = x\log 5 - \log 5$$
$$\log 5 - 2\log 2 = x\log 5 - x\log 2$$
$$\log\left(\frac{5}{2^2}\right) = x(\log 5 - \log 2) = x\log\left(\frac{5}{2}\right)$$
$$\log 1.25 = x \cdot \log 2.5$$
$$\mathbf{x = \frac{log1.25}{log2.5} \approx 0.2435}$$

Wenn Sie die Gleichung anders ordnen, erhalten Sie: $x = \frac{\log 0.8}{\log 0.4}$

e)
$$\sqrt[x]{20} = 5$$
$$20^{\frac{1}{x}} = 5$$
$$\log\left(20^{\frac{1}{x}}\right) = \log 5$$
$$\frac{1}{x} \cdot \log 20 = \log 5 \quad \Big| \cdot x : \log 5$$
$$\mathbf{x = \frac{log20}{log5} \approx 1.86}$$

d)
$$\frac{1}{3^x} = 4$$
$$3^{-x} = 4$$
$$\log(3^{-x}) = \log 4$$
$$(-x)\log 3 = \log 4$$
$$\mathbf{x = -\frac{log4}{log3} \approx -1.262}$$

f)
$$3^{x-1} \cdot 2^{2x} = 5^{3x+1}$$
$$\log(3^{x-1} \cdot 2^{2x}) = \log(5^{3x+1})$$
$$\log(3^{x-1}) + \log(2^{2x}) = \log(5^{3x+1})$$
$$(x-1)\log 3 + 2x\log 2 = (3x+1)\log 5$$
$$x\log 3 - \log 3 + 2x\log 2 = 3x\log 5 + \log 5$$
$$x\log 3 - \log 3 + x\log 2^2 = x\log 5^3 + \log 5$$
$$x\log 3 - \log 3 + x\log 4 = x\log 125 + \log 5$$
$$x\log 3 + x\log 4 - x\log 125 = \log 5 + \log 3$$
$$x\left(\log 3 + \log 4 - \log 125\right) = \log 15$$
$$x\left(\log 12 - \log 125\right) = \log 15$$
$$\mathbf{x = \frac{log15}{log12 - log125} \approx -1.1556}$$