$\cos x + \cos(x + 120°) + \cos(x + 240°) =$

Vorüberlegungen:

$$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 120° = -\cos 60° = -\frac{1}{2}$$

$$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$$

$$\cos 240° = -\cos 60° = -\frac{1}{2}$$

$$\cos(x + 120°) = \cos x \cos 120° - \sin x \sin 120° = \cos x \cdot \left(-\frac{1}{2}\right) - \sin x \cdot \frac{\sqrt{3}}{2}$$

$$\cos(x + 240°) = \cos x \cos 240° - \sin x \sin 240° = \cos x \cdot \left(-\frac{1}{2}\right) - \sin x \cdot \left(-\frac{\sqrt{3}}{2}\right)$$

das ergibt:

$$\cos x + \cos(x + 120°) + \cos(x + 240°) = \cos x + \left(-\frac{\cos x}{2} - \frac{\sqrt{3}\sin x}{2}\right) + \left(-\frac{\cos x}{2} + \frac{\sqrt{3}\sin x}{2}\right) = 0$$